CAAM 336

DIFFERENTIAL EQUATIONS IN SCIENCE AND ENGINEERING

Fall 2012 · Rice University

Lectures:	Section 1: MWF 1PM, Duncan Hall 1070 Section 2: MWF 1PM, Keith-Weiss Geological Lab 100
Web Site:	http://www.caam.rice.edu/~caam336
Instructors:	Section 1: Mark Embree (embree@rice.edu), Abercrombie A101, (713) 348-6160 Office hours: Tuesday 2–4PM in Duncan Hall 3092 Friday 11AM–12PM in Abercrombie A101, or by appointment Section 2: Richard Rankin (Richard.A.Rankin@rice.edu), Duncan Hall 3018, (713) 348-2049 Office hours: Tuesday 3–4PM in Duncan Hall 3092 Thursday 4–5PM in Duncan Hall 3018, or by appointment
Teaching Assistant:	Xin Yang (Xin.Yang@rice.edu) Duncan Hall 3017 Office hours: Monday 4-5PM, Tuesday 4-5PM, both in Duncan Hall 3110, or by appointment
Recitations:	Monday, 7:00-9:00PM, Anderson Biological Lab 123 (starting 27 August)

Grade Policy:	60% exams, 40% homework Class participation will influence borderline and potential A+ grades. Improving performance over the course of the semester will also be considered.
Absence Policy:	Students are strongly encouraged to contribute to our class community by attending and participating in lectures and active learning activities.
Required Text:	Mark S. Gockenbach, *Partial Differential Equations: Analytical and Numerical Methods* Second edition, SIAM, Philadelphia, 2010. (The first edition is also acceptable.)
Exams:	Two take-home, four-hour, closed-book exams will each account for 30% of the final grade. Each of these pledged exams must be your individual, unassisted effort. The first exam will be given in mid-October, the second during the final exam period.
Problem Sets:	Problem sets will be assigned roughly once a week, due at 5pm on the specified date. Mathematically rigorous solutions are expected; strive for clarity and elegance. You may collaborate on the problems, but your write-up must be your own independent work. Transcribed solutions and copied MATLAB code are both unacceptable. *You may not consult solutions from previous sections of this class.* Unless it is specified that a particular calculation must be performed 'by hand,' you are encouraged to use MATLAB's Symbolic Math Toolbox (or Mathematica/Wolfram Alpha/Maple) to compute and simplify tedious integrals and derivatives on the problem sets. As always, you must document your calculations clearly.
Late Policy:	You may turn in two homework assignments one class period late without penalty. Subsequent late assignments will be penalized 20% each. Homework will not be accepted more than one class period late. (This implies that you *may not* use two 'lates' on one assignment.) In exceptional circumstances, please contact the instructor as soon as possible: we adhere to Student Health's 'No Note' policy.
Programming:	Most homework assignments will require a modest amount of MATLAB programming. Your solutions should adhere to good programming standards, and must not be copied from another student. Consult the course web site for pointers to MATLAB resources. You might also consult: D. J. Higham & N. J. Higham, *MATLAB Guide*, 2nd ed., SIAM, Philadelphia, 2005.

Prerequisites:	CAAM 210 (Introduction to Engineering Computation) and MATH 212 (Multivariable Calculus), or permission of the instructor. Less formally: you should be able to write elementary MATLAB programs, and be comfortable with vector calculus and basic vector and matrix operations.
Course Objectives:	CAAM 336 students learn to identify partial differential equations and solve canonical cases using both analytical and computational techniques.
Course Outcomes:	Upon completing this course, students should be able to: 1) explain the concept and computation of a best approximation from a subspace; 2) identify linear operators and compute their spectra in basic cases; 3) apply these concepts to obtain exact series solutions to differential equations; 4) develop finite element approximations to the exact solutions; 5) assess the stability requirement for time-dependent approximations; 6) create MATLAB programs that compute and visualize series and finite element solutions.

Any student with a disability requiring accommodation in this course is encouraged to contact the instructor during the first week of class, and also to contact Disability Support Services in the Ley Student Center.

CAAM 336 · Manifesto and Outline

Differential equations form the heart of applied mathematics: they capture an amazing variety of phenomena in fields ranging from physical science and engineering to biology, from financial derivatives to traffic flow. As useful as these equations are, in practice they can be quite difficult to solve: as you read these words, many of the world's fastest computers are busily calculating solutions to such equations. Though the roots of this field reach back to the 18th century, many important, fascinating problems remain for you to tackle. Nearly all disciplines in science and engineering rely fundamentally on differential equations.

This course focuses on the three most famous partial differential equations: the Poisson (diffusion) equation, the heat equation, and the wave equation. There are two main approaches to solving these equations: one that constructs the exact solution as an infinite series (the 'spectral method') and another that uses numerical methods to quickly build approximations to the solution. For historical reasons these two approaches are usually taught in distinct courses—an unfortunate fissure that masks beautiful and deep connections. This course will emphasize these connections. By mastering these ideas you will not only learn elegant techniques for solving classical problems; you will also develop tools to solve the problems you will discover for yourselves as professional scientists and engineers.

1. Overview and classification of ordinary and partial differential equations
2. Derivation of PDEs from physical models
3. Linear operators (matrices, the Laplacian)
4. Eigenfunctions of the Laplacian; Fourier series
5. Poisson equation: the spectral method
6. Galerkin approximation and the finite element method
7. Poisson equation: the finite element method
8. Heat equation: the spectral method
9. Heat equation: the finite element method; stability
10. Wave equation: D'Alembert's solution; the spectral method
11. Wave equation: finite elements and other numerical methods
12. Special topics: nonlinear problems, multidimensional problems, ...